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A Convergent Nonequilibrium Statistical Mechanical Theory for Dense Gases. II. Transport Coefficients to First Order in the Density

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Using the two-body distribution function found earlier by the authors with the aid of new boundary conditions, the kinetic equation and the transport coefficients are obtained to zeroth and first order in the density. To zeroth order we recover the Boltzmann kinetic equation. To first order the resulting expressions differ from the ones obtained by Choh and Uhlenbeck, due to effects of the medium.

KEY WORDS: Convergent virial expansion of transport coefficients; dense gases; boundary conditions; heat conductivity; shear viscosity; bulk viscosity.

1. INTRODUCTION

In earlier papers[1,2,3] the authors have obtained the first terms in the density expansion of the two-body distribution function of a dense gas using boundary conditions different from those proposed by Bogolyubov. It was found that this virial expansion exists.

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[3] Reference 2 will be referred to as I. Here we use the same notation as in I.

It is the purpose of this paper to use the results found in I in order to obtain the corresponding kinetic equation and transport coefficients to zeroth and first order in the density.

To zeroth order in the density we recover the well-known Boltzmann kinetic equation.

To first order in the density we find results that are different from those obtained by Choh and Uhlenbeck.[3] This difference is due to the fact that we start from the convergent density expansion of the two-body distribution function, obtained in I, which is different from the function used by Choh and Uhlenbeck. These authors use the function obtained by Bogolyubov.[4] The difference in the distribution functions resides in the boundary conditions used to solve the BBGKY hierarchy. In our case we take into account the medium, whereas Bogolyubov did not.

In order to evaluate the transport coefficients, we will use general formal results obtained without using the explicit expression for the two-body distribution function.[5]

In a subsequent paper we will present the calculations leading to the transport coefficients up to second order in the density. It will be shown that these expressions exist.

In Section 2 we give the basic expressions needed for the calculation of the kinetic equation and the transport coefficients. In Section 3 we discuss the approximation to zeroth order in the density and show that the Boltzmann kinetic equation is obtained. In Section 4 we discuss the solution of the kinetic equation to first order in the density. In Section 5 we give the explicit general expressions for the transport coefficients to first order in the density. Finally, in Section 6 we sum up our results.

2. BASIC EXPRESSIONS

In this section we write down several expressions which will be useful for our purpose.

The kinetic equation is obtained from the first equation of the BBGKY hierarchy,

$$\frac{\partial F_1}{\partial t} + \frac{\mathbf{p}}{m} \cdot \frac{\partial F_1}{\partial \mathbf{q}} = \Phi(x \mid F_1) \tag{1}$$

where

$$\Phi(x \mid F_1) = n \int dx_2 \, \theta_{12} F_2(x, x_2 \mid F_1) \tag{2}$$

Here n is the particle density, $x \equiv (\mathbf{p}, \mathbf{q})$, and F_2 is the two-body distribution function as a time-independent functional of F_1.

As usual, we only consider the kinetic equation up to the first order in the gradients of the system. The kinetic equation then becomes[5]

$$\frac{\partial F_1}{\partial t} + \frac{\mathbf{p}}{m} \cdot \frac{\partial F_1}{\partial \mathbf{q}} = \Phi(x \mid F_1(\mathbf{q})) + \int dx'\, \Phi'(x, x' \mid F_1(\mathbf{q}))(\mathbf{q}' - \mathbf{q}) \cdot \left(\frac{\partial F_1}{\partial \mathbf{q}'}\right)_{\mathbf{q}'=\mathbf{q}} \tag{3}$$

where $\Phi(x \mid F_1(\mathbf{q}))$ is evaluated at the local distribution function $F_1(\mathbf{q})$ and $\Phi'(x, x' \mid F_1(\mathbf{q}))$ denotes the functional derivative of Φ with respect to F_1 taken at the point x' and evaluated for the local distribution function $F_1(\mathbf{q})$.

Using the Chapman–Enskog method to solve Eq. (3), with the introduction of the perturbation function ϕ by means of

$$F_1 = F_1^e(1 + \phi) \tag{4}$$

where ϕ represents the linear nonuniformities in the macroscopic variables and F_1^e the local equilibrium single distribution function, one obtains a unique solution in the form

$$\phi = \mathscr{G}(\mathscr{P}^2)\mathscr{P} \cdot \frac{\partial \ln \theta}{\partial \mathbf{q}} + \mathscr{A}(\mathscr{P}^2)\, \mathscr{P}^\circ\mathscr{P} : \frac{\partial \mathbf{u}}{\partial \mathbf{q}} + \mathscr{B}(\mathscr{P}^2) \frac{\partial}{\partial \mathbf{q}} \cdot \mathbf{u} \tag{5}$$

The notation used is explained in Ref. 5. The functions $\mathscr{G}$, $\mathscr{A}$, and $\mathscr{B}$ satisfy certain integral equations[5] whose kernels contain the function F_2. Having determined the functions $\mathscr{G}$, $\mathscr{A}$, and $\mathscr{B}$, one can then obtain the transport coefficients.

It should be mentioned that the results quoted above were obtained without any recourse to a density expansion. Therefore they are valid to all orders in the density.

If we now make a density expansion of F_2

$$F_2(\cdots \mid F_1) = \sum_{l=0}^{\infty} n^l F_2^{(l)}(\cdots \mid F_1) \tag{6}$$

where the first $F_2^{(l)}$ were explicitly obtained in I, one obtains density expansions for Φ and the functions $\mathscr{G}$, $\mathscr{A}$, and $\mathscr{B}$ as follows:

$$\Phi = \sum_{l=1}^{\infty} n^l \Phi^{(l)} \tag{7}$$

$$\mathscr{G} = (1/n)\, \mathscr{G}_B + \mathscr{G}_0 + n\mathscr{G}_1 + \cdots \tag{8}$$

$$\mathscr{A} = (1/n)\, \mathscr{A}_B + \mathscr{A}_0 + n\mathscr{A}_1 + \cdots \tag{9}$$

and

$$\mathscr{B} = (1/n)\, \mathscr{B}_B + \mathscr{B}_0 + n\mathscr{B}_1 + \cdots \tag{10}$$

The various functions appearing in the right-hand sides of Eqs. (7)–(10) satisfy certain integral equations which we will discuss in the following sections.

3. KINETIC EQUATION AND TRANSPORT COEFFICIENTS TO ZEROTH ORDER IN THE DENSITY

In order to find the kinetic equation to zeroth order in the density, one uses the value of $F_2^{(0)}$ obtained in I, namely

$$F_2^{(0)}(x_1, x_2 \mid F_1) = \Gamma_2(x_1, x_2)\, \mathscr{S}_2(x_1, x_2) \prod_{i=1}^{2} F_1(x_i; t) \tag{11}$$

Substituting this expression into Eq. (2), comparing with Eq. (7), and substituting into Eq. (1), one finds that the kinetic equation to zeroth order in the density has the following form:

$$\frac{\partial F_1}{\partial t} + \frac{\mathbf{p}}{m} \cdot \frac{\partial F_1}{\partial \mathbf{q}} = \int dx_2\, \theta_{12} \Gamma_2(\mathbf{q}, \mathbf{q}_2)\, \mathscr{S}_2(\mathbf{p}, \mathbf{p}_2)\, F_1(\mathbf{q}, \mathbf{p}; t)\, F_1(\mathbf{q}, \mathbf{p}_2; t) \tag{12}$$

In this expression the operator $\mathscr{S}_2$, as is indicated, now acts only on the momenta. Proceeding as usual, we find that the right-hand side of Eq. (12) can be written as follows:

$$\text{r.h.s.} = \int d\mathbf{p}_2\, g \int db\, b \int d\epsilon \int_{-\infty}^{\infty} dl\, \Gamma_2(b, l)(\partial/\partial l)[F_1(\mathbf{q}, \mathbf{P}_1; t)\, F_1(\mathbf{q}, \mathbf{P}_2; t)] \tag{13}$$

Here the $\mathbf{q}_2$-integration was changed to an $\mathbf{r}_{21} = \mathbf{q}_2 - \mathbf{q}_1$ integration, and this was done in cylindrical coordinates with the third axis in the direction of the relative velocity $\mathbf{g} = (\mathbf{p}_2 - \mathbf{p}_1)/m$. The coordinate along the third axis is denoted by l and the polar coordinates perpendicular to this axis by b and ϵ. In (13)

$$\mathbf{P}_i = \mathscr{S}_2 \mathbf{p}_i, \qquad i = 1, 2 \tag{14}$$

The integration over l can be written as

$$\int_{-\infty}^{\infty} dl\, \Gamma_2(b, l)(\partial/\partial l)[\cdots] = \int_{-l_0}^{l_0} dl(\partial/\partial l)[\cdots] = [\cdots]_{l=-l_0}^{l=l_0}$$

where l_0 is the correlation length to zeroth order in the density. Finally, one obtains that the kinetic equation to zeroth order in the density is

$$\frac{\partial F_1}{\partial t} + \frac{\mathbf{p}}{m} \cdot \frac{\partial F_1}{\partial \mathbf{q}} = \int d\mathbf{p}_2\, g \int db\, b \int d\epsilon \times [F_1(\mathbf{q}, \mathbf{p}_i'; t)\, F_1(\mathbf{q}, \mathbf{p}_2'; t) - F_1(\mathbf{q}, \mathbf{p}; t)\, F_1(\mathbf{q}, \mathbf{p}_2; t)] \tag{15}$$

where $\mathbf{p}_i'$ denotes the momenta after the collision. Equation (15) is the usual Boltzmann equation. It should be mentioned that the potentials we are considering are strongly repulsive ones.

As is well known, the solution of Eq. (15), to linear terms in the gradient, is

$$F_1 = F_1^e(1 + \phi_B) \tag{16}$$

with

$$\phi_B = \mathscr{G}_B \mathscr{P} \cdot [\partial(ln\theta)/\partial \mathbf{q}] + \mathscr{A}_B \mathscr{P}^\circ \mathscr{P} : \partial \mathbf{u}/\partial \mathbf{q} \tag{17}$$

where $\mathscr{G}_B$ and $\mathscr{A}_B$ are solutions to well-known integral equations.[6] The transport coefficients obtained from Eq. (17) have already been obtained for different intermolecular potentials[6] and found to be independent of density.

We have thus shown that using the new boundary conditions discussed in I in order to solve the BBGKY hierarchy, we recover the usual Boltzmann equation for the dilute gas and the corresponding well-known transport coefficients to zeroth order in the density.

4. SOLUTION OF THE KINETIC EQUATION TO FIRST ORDER IN THE DENSITY

To first order in the density the solution of the kinetic equation is[5]

$$\phi_0 = \mathscr{G}_0 \mathscr{P} \cdot [\partial(\ln \theta)/\partial \mathbf{q}] + \mathscr{A}_0 \mathscr{P}^\circ \mathscr{P} : (\partial \mathbf{u}/\partial \mathbf{q}) + \mathscr{B}_0(\partial/\partial \mathbf{q}) \cdot \mathbf{u} \tag{18}$$

where the functions $\mathscr{G}_0$, $\mathscr{A}_0$, and $\mathscr{B}_0$ satisfy the following set of integral equations:

$$F_1^e \frac{\mathscr{P}}{m} \frac{d}{d\theta} [\theta B_1(\theta)] \frac{n}{2} - \int dx' \, \Phi'^{(0)}(x, x' \mid F_1^e(\mathbf{q})) F_1^e(\mathbf{p}') \left(\frac{\mathscr{P}'^2}{2m\theta} - \frac{3}{2} \right) (\mathbf{q}' - \mathbf{q})$$

$$+ \int d\mathbf{p}' \, \Phi'^{(1)}(x, \mathbf{p}' \mid F_1^e(\mathbf{q})) F_1^e(\mathbf{p}') \mathscr{P}' \mathscr{G}_B$$

$$= \int d\mathbf{p}' \, \Phi'^{(0)}(x, \mathbf{p}' \mid F_1^e) \mathscr{P}_1' F_1^e(\mathbf{p}') \mathscr{G}_0 \tag{19a}$$

$$\int d\mathbf{p}' \, \Phi'^{(1)}(x, \mathbf{p}' \mid F_1^e(\mathbf{q})) F_1^e(\mathbf{p}') \mathscr{P}'^\circ \mathscr{P}' \mathscr{A}_B$$

$$- \frac{1}{\theta} \int dx' \, \Phi'^{(0)}(x, x' \mid F_1^e(\mathbf{q})) S^\circ S F_1^e(\mathbf{p}')$$

$$= - \int d\mathbf{p}' \, \Phi'^{(0)}(x, \mathbf{p}' \mid F_1^e(\mathbf{q})) F_1^e(\mathbf{p}') \mathscr{P}'^\circ \mathscr{P}' \mathscr{A}_0 \tag{19b}$$

$$F_1^e \left\{ \left(1 - \frac{\mathscr{P}^2}{3m\theta}\right)\left[\frac{n}{2} B_1(\theta) - \frac{C_1^{(0)}}{n\theta}\right] \right.$$
$$\left. - \frac{1}{3\theta} \int dx'\, \Phi'^{(0)}(x, x' \mid F_1^e(\mathbf{q}))\, \mathscr{P}' \cdot (\mathbf{q}' - \mathbf{q})\, F_1^e(\mathbf{p}') \right\}$$
$$= - \int d\mathbf{p}'\, \Phi'^{(0)}(x, \mathbf{p}' \mid F_1^e(\mathbf{q}))\, F_1^e(\mathbf{p}')\, \mathscr{B}_0 \tag{19c}$$

with the subsidiary conditions given by

$$\int \begin{Bmatrix} \mathscr{G}_0 \\ \mathscr{B}_0 \end{Bmatrix} p^2 F_1^e(\mathbf{p})\, d\mathbf{p} = 0 \tag{20}$$

Using Eqs. (26) and (29) of I for $F_2^{(0)}$ and $F_2^{(1)}$, respectively, one obtains that the functional derivatives of Φ with respect to F_1 are, to zeroth order in the density,

$$\Phi'^{(0)}(x_1, x' \mid F_1^e)\, F_1^e(\mathbf{p}')$$
$$= \int dx_2\, \theta_{12} \Gamma_2(\mathbf{q}_1, \mathbf{q}_2)\, \mathscr{S}_2(x_1, x_2)\, F_1^e(x_1)\, F_1^e(x_2)[\delta(x_1 - x') + \delta(x_2 - x')] \tag{21}$$

$$\Phi'^{(0)}(x_1, \mathbf{p}' \mid F_1^e)\, F_1^e(\mathbf{p}')$$
$$= \int dx_2\, \theta_{12} \Gamma_2(\mathbf{q}_1, \mathbf{q}_2)\, \mathscr{S}_2(x_1, x_2)\, F_1^e(x_1)\, F_1^e(x_2)[\delta(\mathbf{p}_1 - \mathbf{p}') + \delta(\mathbf{p}_2 - \mathbf{p}')] \tag{22}$$

and to first order in the density,

$$\Phi'^{(1)}(x_1, \mathbf{p}' \mid F_1^e)\, F_1^e(\mathbf{p}') = \int dx_2\, \theta_{12} \{ g_2^{(1)}(\mathbf{q}_1, \mathbf{q}_2)\, \mathscr{S}_2(x_1, x_2)\, F_1^e(x_1)\, F_1^e(x_2)$$
$$\times [\delta(\mathbf{p}_1 - \mathbf{p}') + \delta(\mathbf{p}_2 - \mathbf{p}')] \}$$
$$+ \int dx_3\, \mathscr{O}_3(x_1, x_2, x_3)\, F_1^e(x_1)\, F_1^e(x_2)\, F_1^e(x_3)$$
$$\times [\delta(\mathbf{p}_1 - \mathbf{p}') + \delta(\mathbf{p}_2 - \mathbf{p}') + \delta(\mathbf{p}_3 - \mathbf{p}')] \tag{23}$$

The operator $\mathscr{O}(x_1, x_2, x_3)$ is defined in I.

Substituting Eqs. (21)–(23) into Eqs. (19) and performing the usual calculations, one finds that the equation for $\mathscr{G}_0$ has the form

$$X(\mathbf{p}) \frac{\mathscr{P}}{m} \left[\frac{d}{d\theta}(\theta B_1) + B_1\left(\frac{\mathscr{P}^2}{2m\theta} - \frac{5}{2}\right) + \bar{\beta}(\mathbf{p}) + \bar{L}(\mathbf{p}) + \bar{A}(\mathbf{p})\right] = C_B(\mathscr{P}\mathscr{G}_0) \tag{24}$$

where

$$X(\mathbf{p}_1)(\mathscr{P}_1/m)\,\bar{\beta}(\mathbf{p}_1)$$
$$= \int dx_2\,\theta_{12}g_2^{(1)}(\mathbf{q}_1,\mathbf{q}_2)\,\mathscr{S}_2(x_1,x_2)\,X(\mathbf{p}_1)\,X(\mathbf{p}_2)\sum_{i=1}^{2}\mathscr{P}_i\mathscr{G}_B(\mathscr{P}_i^2) \tag{25}$$

$$X(\mathbf{p}_1)(\mathscr{P}_1/m)\,\bar{A}(\mathbf{p}_1)$$
$$= \int dx_2\,\theta_{12}\int dx_3\,\mathscr{O}_3(x_1,x_2,x_3)\,X(\mathbf{p}_1)\,X(\mathbf{p}_2)\,X(\mathbf{p}_3)\sum_{i=1}^{3}\mathscr{P}_i\mathscr{G}_B(\mathscr{P}_i^2) \tag{26}$$

and

$$X(\mathbf{p}_1)(\mathscr{P}_1/m)\,\bar{L}(\mathbf{p}_1)$$
$$= \int dx_2\,\theta_{12}\Gamma_2(\mathbf{q}_1,\mathbf{q}_2)\,\mathscr{S}_2(x_1,x_2)(\mathbf{r}/2)[(\mathscr{P}_2^2-\mathscr{P}_1^2)/2m\theta]\,X(\mathbf{p}_1)\,X(\mathbf{p}_2) \tag{27}$$

$$F_1^e(\mathbf{p}) = nX(\mathbf{p}) \tag{28}$$

The operator C_B appearing in Eq. (24) is the usual linearized Boltzmann operator given by[6]

$$C_B(h) = \int\int d\mathbf{p}_2\,d\Omega\,I(g,\epsilon)\,X(\mathbf{p}_2)[h(\mathbf{p}_1')+h(\mathbf{p}_2')-h(\mathbf{p}_1)-h(\mathbf{p}_2)] \tag{29}$$

In Eq. (24) B_1 is the second virial coefficient.

The function $\mathscr{G}_0$ obtained from Eq. (24) has to satisfy the subsidiary conditions (20).

The equation for $\mathscr{A}_0$ is given by

$$[X(\mathbf{p})/m\theta]\,\mathscr{P}^\circ\mathscr{P}[B_1+\bar{\Gamma}(\mathbf{p})+\bar{M}(\mathbf{p})+\bar{N}(\mathbf{p})] = C_B(\mathscr{P}^\circ\mathscr{P}\mathscr{A}_0) \tag{30}$$

where

$$[X(\mathbf{p}_1)/m\theta](\mathscr{P}^\circ\mathscr{P})_1\,\bar{M}(\mathbf{p}_1)$$
$$= -\int dx_2\,\theta_{12}g_2^{(1)}(\mathbf{q}_1,\mathbf{q}_2)\,\mathscr{S}_2(x_1,x_2)\,X(\mathbf{p}_1)\,X(\mathbf{p}_2)\sum_{i=1}^{2}(\mathscr{P}^\circ\mathscr{P})_i\,\mathscr{A}_B(\mathscr{P}_i^2) \tag{31}$$

$$(1/m)\,X(\mathbf{p}_1)(\mathscr{P}^\circ\mathscr{P})_1\,[B_1+\bar{\Gamma}(\mathbf{p}_1)]$$
$$= \int dx_2\,\theta_{12}\Gamma_2(\mathbf{q}_1,\mathbf{q}_2)\,\mathscr{S}_2(x_1,x_2)\,X(\mathbf{p}_1)\,X(\mathbf{p}_2)\sum_{i=1}^{2}(S^\circ S)_i \tag{32}$$

and

$$[X(\mathbf{p}_1)/m\theta](\mathscr{P}^\circ\mathscr{P})_1\,\bar{N}(\mathbf{p}_1)$$
$$= -\int dx_2\,\theta_{12}\int dx_3\,\mathscr{O}_3(x_1,x_2,x_3)\,X(\mathbf{p}_1)\,X(\mathbf{p}_2)\,X(\mathbf{p}_3)\sum_{i=1}^{3}(\mathscr{P}^\circ\mathscr{P})_i\,\mathscr{A}_B(\mathscr{P}_i^2) \tag{33}$$

The function $\mathscr{A}_0$ does not have to satisfy any subsidiary condition.

Using the same arguments as given by several authors,[3] we also find that the bulk viscosity vanishes to first order in the density.

One can, in principle, solve the integral equations (24) and (30) for the functions $\mathscr{G}_0$ and $\mathscr{A}_0$, respectively. This requires the use of an intermolecular potential. In forthcoming publications we will present the solutions of these integral equations for different molecular models. For the time being, we will assume that the functions $\mathscr{G}_0$ and $\mathscr{A}_0$ are determined.

5. TRANSPORT COEFFICIENTS TO FIRST ORDER IN THE DENSITY

The transport coefficients to first order in the density were obtained without any reference to the explicit form of F_2.[5] Substituting in these expressions the $F_2^{(1)}$ given by Eq. (29) of I, one finds that the thermal conductivity λ is, to first order in the density,

$$\lambda^{(1)} = \lambda_\kappa^{(1)} + \lambda_{\varphi 1}^{(1)} + \lambda_{\varphi 2}^{(1)} \tag{34}$$

with

$$\lambda_\kappa^{(1)} = -(1/6m^2\theta) \int d\mathbf{p}\, \mathscr{P}^4 X(\mathbf{p})\, \mathscr{G}_0(\mathscr{P}^2) \tag{35}$$

$$\lambda_{\varphi 1}^{(1)} = -(1/12m\theta) \int dx_2\, d\mathbf{p}_1\, \mathbf{r} \cdot (\mathscr{P}_1 + \mathscr{P}_2)[\varphi'(r)/r]\, \Gamma_2(\mathbf{q}_1, \mathbf{q}_2)$$

$$\times\, \mathscr{S}_2(x_1, x_2)\, X(\mathbf{p}_1)\, X(\mathbf{p}_2)\mathbf{r} \cdot \sum_{i=1}^{2} \mathscr{P}_i \mathscr{G}_B(\mathscr{P}_i^2) \tag{36}$$

and

$$\lambda_{\varphi 2}^{(1)} = (1/6m\theta) \int dx_2\, d\mathbf{p}_1\, \varphi(r)\, \Gamma_2(\mathbf{q}_1, \mathbf{q}_2)$$

$$\times\, \mathscr{P}_1 \cdot \mathscr{S}_2(x_1, x_2)\, X(\mathbf{p}_1)\, X(\mathbf{p}_2) \sum_{i=1}^{2} \mathscr{P}_i \mathscr{G}_B(\mathscr{P}_i^2) \tag{37}$$

Analogously, the shear viscosity η to first order in the density is given by

$$\eta^{(1)} = \eta_\kappa^{(1)} + \eta_\varphi^{(1)} \tag{38}$$

with

$$\eta_\kappa^{(1)} = -(1/15m) \int d\mathbf{p}\, \mathscr{P}^4 X(\mathbf{p})\, \mathscr{A}_0(\mathscr{P}^2) \tag{39}$$

$$\eta_\varphi^{(1)} = -(1/20) \int dx_2\, d\mathbf{p}_1 [\varphi'(r)/r]\, \Gamma_2(\mathbf{q}_1, \mathbf{q}_2) \times \mathscr{S}_2(x_1, x_2)\, X(\mathbf{p}_1)\, X(\mathbf{p}_2) \sum_{i=1}^{2} [(\mathbf{r} \cdot \mathscr{P}_i)^2 - \tfrac{1}{3} r^2 \mathscr{P}_i^2]\, \mathscr{A}_B(\mathscr{P}_i^2) \tag{40}$$

The bulk viscosity, to first order in the density, vanishes.

6. CONCLUSIONS

In this work we have shown, that using the results obtained in I with the new boundary conditions, one recovers the usual Boltzmann kinetic equation for the dilute gas.

We also give general expressions for the transport coefficients to first order in the density.

It can be seen from the results in Section 4 and 5 that the transport coefficients to first order in the density are different from the results obtained by Choh and Uhlenbeck[3] using the boundary conditions proposed by Bogolyubov. In fact, first of all, the integral equations for $\mathscr{G}_0$ and $\mathscr{A}_0$ [Eqs. (24) and (30)] have the same formal structure as the equations obtained by Choh and Uhlenbeck. However, the left-hand sides of the equations are different. The difference lies in the fact that in our expressions the medium is taken into account, and appears through the factors Γ_s and $g_s^{(l)}$.

Moreover, the integrands of the expressions for the transport coefficients [Eqs. (34) and (38)] also differ from the ones obtained by Choh and Uhlenbeck, for the same reason as given above.

Finally, we would like to mention that we have obtained general expressions for the transport coefficients to second order in the density, and these will be presented in a subsequent publication.

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